STUDENT ID NO

MULTIMEDIA UNIVERSITY

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FINAL EXAMINATION

TRIMESTER 1, 2018/2019

TMA1301 – COMPUTATIONAL METHODS

(TC01 / TT01)

25 OCTOBER 2018
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 5 pages (inclusive of the cover page).
2. Answer **ALL** questions. The distribution of marks for each question is given.
3. The **Appendix** section begins from **page 4 onwards**.
4. Please write all your answers in the Answer Booklet provided.
5. Please write the answers for each question on a new page of your Answer Booklet.

QUESTION 1: ROOTS OF EQUATIONS **[Total Marks: 7]**

Given an equation as follows,

$$f(x) = \sin x + e^x$$

Using an initial value of $a = 0$, apply Newton's method to find a root for the function with a tolerance of 1×10^{-5}. Write your answers in 5 decimal places. [7 marks]

QUESTION 2: NUMERICAL INTEGRATION **[Total Marks: 8]**

Given an integral as follows,

$$\int_0^{0.5} \sqrt{2x+1}\, dx$$

(a) Use the Simpson's Rule with 4 equal parts ($n = 4$) to evaluate this integral. Write your answers in 6 decimal places. [7 marks]

(b) Given that the actual value is 0.609476, calculate the relative error between the actual value and your answer in (a). [1 mark]

QUESTION 3: LEAST SQUARES PROBLEMS, INTERPOLATION AND POLYNOMIAL APPROXIMATION **[Total Marks: 10]**

Given the following data sets,

x	-4	-1.5	0	2	5	8
y	-20	-1	2	12	42	76

(a) Find the best fit line of $y = a + bx$ using the method of least squares. Write your answers in 3 decimal places. [9 marks]

(b) Using your answer in (a), find y for $x = 10$. Write your answer in 3 decimal places. [1 mark]

Continued...

QUESTION 4: MATRICES AND SYSTEMS OF LINEAR EQUATIONS

[Total Marks: 15]

(a) Given the following systems of linear equations,

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solve for x, y, z using the method of Gaussian Elimination with Back Substitution.

[7 marks]

(b) For the following systems of linear equations,

$$3u + v = -2$$
$$u + 8v = 1$$

(i) Find the values of u and v using both **Jacobi Method** and **Gauss-Siedel Method with 2 iterations**. Begins your iteration with $u_0 = 0, v_0 = 0$. <u>Round off your answers to 6 decimal places.</u>

(ii) Calculate the absolute error between the two methods for each u and v that you get in (i).

[6 + 2 = 8 marks]

End of Questions.

APPENDIX: USEFUL FORMULAS

ROOTS OF EQUATION

Bisection Method	Secant Method	Newton-Raphson Method
$p_n = \dfrac{b_n + a_n}{2}$	$p_{n+1} = p_n - \dfrac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$	$p_{n+1} = p_n - \dfrac{f(p_n)}{f'(p_n)}$

NUMERICAL INTEGRATION

Trapezoidal Rule

$$
\begin{aligned}
A_T &= \int_a^b f(x)\,dx \\
&\approx \frac{h}{2}\left\{f(x_1) + f(x_{n+1}) + 2\left[f(x_2) + f(x_3) + \ldots + f(x_n)\right]\right\}
\end{aligned}
$$

where $x_n = a + (n-1)h,\ n = 1, 2, \ldots$

Simpson's Rule

$$
\begin{aligned}
A_T &= \int_a^b f(x)\,dx \\
&\approx \frac{h}{3}\left\{\begin{array}{l} f(x_1) + f(x_{2n+1}) + 4\left[f(x_2) + f(x_4) + \ldots + f(x_{2n})\right] \\ +2\left[f(x_3) + f(x_5) + \ldots + f(x_{2n-1})\right] \end{array}\right\}
\end{aligned}
$$

where $x_n = a + (n-1)h,\ n = 1, 2, \ldots$

Romberg Algorithm

$$R(0,0) = \frac{1}{2}(b-a)\left[f(a) + f(b)\right]$$

$$R(n,0) = \frac{1}{2}R(n-1,0) + h\sum_{k=1}^{2^{n-1}} f(a + (2k-1)h);\quad h = \frac{(b-a)}{2^n},\quad n \geq 1$$

$$R(n,m) = R(n,m-1) + \frac{1}{4^m - 1}\left[R(n,m-1) - R(n-1,m-1)\right],$$

where $n \geq 1,\ m \geq 1,\ m \leq n$

LEAST SQUARES PROBLEMS, INTERPOLATION AND POLYNOMIAL APPROXIMATION

Lagrange Coefficient

$$L_i(x)=\frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)}$$

$where\ i=0,1,\dots,n$

Linear Least Squares

$y=a+bx$

$$a=\frac{\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}y_i-\sum_{i=1}^{n}x_iy_i\sum_{i=1}^{n}x_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2} \qquad b=\frac{n\sum_{i=1}^{n}x_iy_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}$$

$y=ae^{bx}$

$$A=\frac{\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}Y_i-\sum_{i=1}^{n}x_iY_i\sum_{i=1}^{n}x_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2} \qquad b=\frac{n\sum_{i=1}^{n}x_iY_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}Y_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}$$

$y=ax^b$

$$A=\frac{\sum_{i=1}^{n}X_i^2\sum_{i=1}^{n}Y_i-\sum_{i=1}^{n}X_iY_i\sum_{i=1}^{n}X_i}{n\sum_{i=1}^{n}X_i^2-\left(\sum_{i=1}^{n}X_i\right)^2} \qquad b=\frac{n\sum_{i=1}^{n}X_iY_i-\sum_{i=1}^{n}X_i\sum_{i=1}^{n}Y_i}{n\sum_{i=1}^{n}X_i^2-\left(\sum_{i=1}^{n}X_i\right)^2}$$

End of Page.